THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH 2070A (First Term, 2018-19)
Algebraic Structures
Course Outline

Outline

This course is intended as an introduction to modern abstract algebra and the algebraic way of thinking in advanced mathematics. The course focuses on basic algebraic concepts which arise in various areas of advanced mathematics, and emphasizes on the underlying algebraic structures which are common to various concrete mathematical examples.

Topics include:

- Group Theory - examples of groups including permutation and dihedral groups, subgroups, the Theorem of Lagrange, group homomorphisms.
- Ring Theory - examples of rings including the ring of integers and polynomial rings, integral domains, fields, ring homomorphisms, ideals and quotient rings.
- Field Theory - examples of field extensions and finite fields.

Prerequisites

Students taking this course should have taken MATH 1010 *University Mathematics* and MATH 1050 *Foundation of Modern Mathematics* (or classes at equivalent level). It would be very helpful if you have also taken MATH 1030 *Linear Algebra I*.

Class Information

- Instructor: CHAN Kwok Wai (Office: LSB 212; Email: `kwchan@math.cuhk.edu.hk`)
- Teaching Assistant: HAU Chun Yin (Office: LSB 232; Email: `cyhau@math.cuhk.edu.hk`)
- Lectures: Mon 2:30pm - 3:15pm at LSB LT4; Wed 2:30pm - 4:15pm at LSB C2
- Tutorials: Mon 3:30pm - 4:15pm at LSB LT4
- Webpage: `https://www.math.cuhk.edu.hk/course/1819/math2070a`

Suggested Texts

- Lecture notes available at the course webpage.
- Artin, *Algebra*, Prentice Hall, 2nd edition.
- Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley, 7th edition (textbook for MATH 3030).

Lectures, Tutorials and Homeworks

Lectures: Students are expected to attend ALL the lectures. The lectures will focus mainly on the theoretical concepts and proofs, supplemented occasionally with some illustrative examples. The chapters in the lecture notes are numbered by weeks. As the lectures would only cover the most essential materials (at a rather fast pace), it would be very helpful if you have read (or at least skimmed through) the relevant chapter beforehand.

Tutorials: Students are expected to attend ALL the tutorials. The tutorials will cover more examples and computational aspects of the materials. You may choose to attend either of the tutorial sections. There will be times during the tutorials for discussions and working out some exercises together. All the materials (except otherwise stated) covered in lectures and tutorials will be covered in the midterms and final exam.

Homeworks: There will be weekly problem sets, usually posted on Mondays and due on Thursday in the following week. Each problem set consists of two parts – the compulsory part and the optional part. You only need to hand in your solutions of the compulsory part. But you are highly recommended to work out the optional part at home as well. Keep in mind that the best way to learn mathematics is to work out exercises and get the feeling by yourself! The full mark for each homework is 1 point. There will be around 12-13 sets of homework and only the best 10 of your problem sets will be counted.

Assessment

- 10%: Homework
- 40%: Midterm (Oct 31st, Wednesday, 2:30pm in class)
- 50%: Final

Tentative Schedule

- Week 1–5: Group theory
  - definition and basic examples of groups
  - cyclic groups, symmetric groups and dihedral groups
  - subgroups, cyclic subgroups and generating sets
  - equivalence relations and partitions; cosets and the Theorem of Lagrange
  - group homomorphisms and isomorphisms, and some basic examples
- Week 6–12: Ring theory
  - definition and basic examples of rings
  - polynomials and polynomial rings
  - integral domains and fields; field of fractions
  - ring homomorphisms and isomorphisms, and some basic examples; subrings and ideals
  - quotient rings
  - factorization of polynomials, Euclidean algorithm, gcd
  - Gauss' Lemma and Eisenstein's Criterion
- Week 13: Field theory
  - basic examples of field extensions and finite fields